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Comments on Higher Derivative Operators in Some SUSY Field Theories

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We study the leading irrelevant operators along the flat directions of certain supersymmetric theories. In particular, we focus on finite $N = 2$ (including $N = 4$) supersymmetric field theories in four dimensions and show that these operators are completely determined by the symmetries of the problem. This shows that they are generated only at one loop and are not renormalized beyond this order. An instanton computation in similar three dimensional theories shows that these terms are renormalized. Hence, the four dimensional non-renormalization theorem of these terms is not valid in three dimensions.

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1. Introduction

Recent advances in the understanding of supersymmetric field theories and string theory were made possible by the special properties of these theories. In particular, the objects which are annihilated by some of the supersymmetries are very special and can often be determined explicitly.

In theories with four supercharges (like the $N=1$ theory in four dimensions) the superpotential is annihilated by half the supercharges and therefore it is supersymmetric after integrating over the other half of superspace. This property is related to the holomorphy of the superpotential. This fact underlies the standard perturbative non-renormalization theorem [1], and often permits an exact, non-perturbative determination of the superpotential (see, e.g. [2]). The Kahler potential in these theories is not annihilated by half the supercharges and therefore it is in general complicated and it is not known how to determine it exactly.

In theories with eight supercharges (like the $N=2$ theory in four dimensions) the superpotential is trivial. Here the Kahler potential is determined by a holomorphic prepotential $\mathcal{F}$ which is annihilated by half the supercharges. The leading terms in the action are given by an integral over the remaining half of superspace [3]

$$\int d^4\theta \mathcal{F}. \tag{1.1}$$

Again, the holomorphy of $\mathcal{F}$ is the key fact which makes it possible to determine it exactly [4,5]. Higher dimension operators in these theories, which are integrated over all of superspace are not controlled by such holomorphy and are in general hard to calculate. For a discussion of such terms see [6,7]. In the next section we will show that in the special case of the finite $N=2$ $SU(2)$ theories these terms can be determined and they are given exactly by the one loop contribution.

In theories with sixteen supercharges (like the $N=4$ theory in four dimensions) there is no simple superspace formalism with a finite number of fields. (A survey of these theories will appear in [8].) Therefore, it is not easy to find the most general supersymmetric action. However, a simple scaling argument (see, e.g. [9]) allows us to organize them. In any number of dimensions we can assign weight -1 to the coordinates X^μ and hence weight

1 to the derivatives. Even if no superspace formalism is known, we should assign weight $\frac{1}{2}$ to the supercharges. Since we plan to study functions on the moduli space, we should assign weight zero to all the scalar moduli. By supersymmetry, this means that we should assign weight $\frac{1}{2}$ to all the fermions and weight zero to all gauge fields. If we use auxiliary fields we assign them higher weights.

With this assignment the superpotential in $N = 1$ theories is integrated over two θ's and leads to terms of weight one. The Kahler potential is integrated over four θ's and leads to terms of weight two. After integrating out the auxiliary fields, terms of lower weight can be generated; e.g. a potential of weight zero is generated after integrating out the auxiliary fields.

It is important to stress that this scaling is not the same as dimensional analysis which is important in the question of renormalizability. For instance, the superpotential leads to terms with two fermions and any number of scalars all of which are of weight one. The Kahler potential leads to various terms with derivatives and if it is not canonical, it can also lead to terms with four fermions and any number of scalars. All these terms are of weight two.

Using this scaling, we expect that the terms which can be controlled in theories with sixteen supercharges are of weight four. This would correspond to an integral over half of superspace, if there were a superspace formulation of the theory. Among these are terms with four space-time derivatives and any number of bosons. They also include terms with two space-time derivatives, any number of bosons and four fermions, and terms with eight fermions and any number of bosons. The goal of this paper is to analyze such terms in three and four dimensions.

In section 3 we analyze these terms in the four dimensional $N = 4$ theory. We argue that they are given exactly by the one loop contribution. We study their potential renormalization by instantons and conclude that instantons cannot renormalize them.

In section 4 we study the $N = 8$ theory in three dimensions where we show that instantons do renormalize these terms. After the completion of this work we received two interesting papers [10,11], which also analyze these terms in greater detail than we do here, but from a different perspective.

Such terms with four space-time derivatives have recently figured in the Matrix model proposal of Banks, Fischler, Shenker and Susskind [12]. (They are also discussed in [13].) For agreement with eleven dimensional supergravity it was suggested that in the quantum mechanics problem with sixteen supercharges these terms are not renormalized beyond one loop. The fact that we find violation of this non-renormalization theorem in three dimensions might suggest that it is also violated in one dimensions (quantum mechanics). Even if this conclusion is correct, it is not clear what consequences this effect has on the Lorentz invariance of the theory of [12].

2. Four derivative terms in $N = 2$ in $d = 4$

In four dimensions, $N = 2$ theories for which the one loop β function vanishes are finite, conformally invariant theories. Example include gauge theories with gauge group $SU(N_c)$ and $2N_c$ hypermultiplets [5]. For these theories, we can easily prove a non-renormalization theorem for the $F^4_{\mu\nu}$ terms. The proof exploits the fact that the couplings of the vector multiplet alone are easily written in $N = 2$ superspace, as well as the scale invariance of the theory.

Consider the case of gauge group $SU(2)$. At low energies the theory has one vector multiplet Ψ. It can be written, in $N = 2$ superspace [3], as

$$\Psi = \Phi + \tilde{\theta}^\alpha W_\alpha + \tilde{\theta}^2 \overline{D}^2 \Phi^\dagger \tag{2.1}$$

where Φ is an $N = 1$ chiral field (a function of the $N = 1$ θ's), W_α is the field strength multiplet (again in $N = 1$ notation), and $\tilde{\theta}$ represent the extra anticommuting coordinates required by $N = 2$. The kinetic terms arise from

$$\int d^2\theta d^2\tilde{\theta} \tau \Psi^2 \tag{2.2}$$

where $\tau = \frac{\theta}{4\pi} + \frac{2\pi i}{g^2}$ is the gauge coupling. Using the scale invariance of the theory, under which Ψ has dimension one, we learn that (2.2) remains quadratic after quantum corrections are included. This leaves open the possibility that the bare coefficient τ is replaced by a function of τ in the effective theory. The analysis of [5] shows that no such

corrections are present if we define τ to be compatible with duality (see also the discussion in [14]).

Terms of weight four arise from couplings of the type

$$\int d^8\theta \mathcal{H}(\Psi, \Psi^\dagger, \tau, \tau^\dagger). \tag{2.3}$$

These must respect all the symmetries of the problem. $\mathcal{H}$ must be dimensionless, while Ψ has dimension one. $\mathcal{H}$ must also respect the $U(1)_R$ symmetry. Because the theory is scale invariant, there can be no scale (other than Ψ itself). For fixed τ there is a unique, non-trivial form permitted by the symmetries:

$$\mathcal{H} \sim \ln(\frac{\Psi}{\Lambda}) \ln(\frac{\Psi^\dagger}{\Lambda}) = \frac{1}{2} \ln^2(\frac{\Psi^\dagger \Psi}{\Lambda^2}) + (f(\Psi) + c.c) \tag{2.4}$$

(the last two terms do not survive integration over superspace, keeping in mind that Ψ is chiral). The scale Λ here is a fake; the terms involving $\ln \Lambda$ are multiplied by chiral or antichiral functions, which vanish when integrated over all of superspace. Similarly, this expression respects the $U(1)_R$ symmetry, since the change in the Lagrangian under such a transformation is an integral over a chiral or antichiral superfield. Note that at general point in the moduli space, this Lagrangian is to be interpreted by expanding the field about that point.

To determine the τ dependence of (2.4) we follow [1] and promote τ to a background superfield. The coupling in (2.2) shows that it must be in a vector superfield. Now, the scale invariance and $U(1)_R$ invariance discussed in the previous paragraph are ruined unless there is no τ dependence at all. This statement implies that there are neither perturbative nor non-perturbative corrections. Thus the $F_{\mu\nu}^4$ terms arise only at one loop in the finite $N = 2$ theories.

3. Four derivative terms in $N = 4$ in $d = 4$

The discussion of the previous section can be applied immediately to the $N = 4$ theories in $d = 4$. These theories can be described in $N = 2$ language. Each $N = 4$ multiplet consists of a vector multiplet of $N = 2$ and a hypermultiplet in the adjoint

representation. Focusing again on $SU(2)$ gauge theory, the light degrees of freedom on the moduli space of vacua are in a single $N = 4$ multiplet. Our choice of $N = 2$ decomposition of this multiplet is such that the scalar expectation value is in the vector of $N = 2$ and the $N = 2$ hypermultiplet does not have an expectation value. Repeating the arguments above gives again a unique form for the terms in the effective action involving the vector fields, Ψ, alone

$$\mathcal{H} \sim \ln(\frac{\Psi}{\Lambda}) \ln(\frac{\Psi^\dagger}{\Lambda}). \tag{3.1}$$

Just as before, one can argue that this term is not renormalized.

It is useful to verify that the action indeed has this structure. The one loop computation is a straightforward Feynman diagram calculation. To determine the expected form of the action, one expands

$$\Psi = v + \Psi', \tag{3.2}$$

where Ψ' is the fluctuating field. Then one can develop eqn. (3.1) in powers of Ψ'. This yields (dropping terms which vanish upon integration over all of $N = 2$ superspace):

$$\mathcal{H} \sim \frac{1}{v^2}\Psi^\dagger\Psi - \frac{1}{2v^3}(\Psi^\dagger\Psi^2 + \Psi^{\dagger 2}\Psi) + \frac{1}{3v^4}(\Psi^\dagger\Psi^3 + \Psi^{\dagger 3}\Psi) + \frac{1}{4v^4}\Psi^{\dagger 2}\Psi^2 + \mathcal{O}(\Psi^5). \tag{3.3}$$

This expression can readily be rewritten in $N = 1$ language, and then in terms of component fields. Ref. [7] gives a general expression for the integral of $\mathcal{H}$, eqn. (2.3), in terms of $N = 1$ superfields. In particular, there are terms

$$\frac{1}{v^2}\int d^4\theta[-2\nabla^{\alpha\dot\alpha}\Phi^*\nabla\delta_{\alpha\dot\alpha}\Phi + 4i\bar W^{\dot\alpha}\nabla^{\alpha}_{\dot\alpha}W_\alpha + \frac{1}{v^2}W^2_\alpha \bar W^2_{\dot\alpha} + \ldots] \tag{3.4}$$

(we are using the notation of ref. [7]). The remaining θ integrals yield a variety of component interactions. For example, there is a four-gaugino interaction,

$$\frac{4}{v^2}\lambda\sigma^\mu\partial_\mu\lambda^*\lambda^*\bar\sigma^\mu\partial_\mu\lambda. \tag{3.5}$$

Similarly, there is a two gaugino term, with coefficient

$$\frac{8}{v^2}\lambda^*\partial^2\partial_\mu\lambda. \tag{3.6}$$

We have verified that these (and other) terms are generated with the correct coefficients, by performing a calculation in the 't Hooft-Feynman gauge.

While this calculation is completely straightforward, it is perhaps worth describing a few of its features. The two gaugino term can simply be read off of the one loop propagator, expanding to order p^3. On finds

$$\frac{8}{3}\frac{g^2}{16\pi^2 M^2}p^2 \not{p} \tag{3.7}$$

where M^2 is the vector meson mass. For the four fermion operators, there are many terms. One can simplify the computation by taking a simple choice of the external momenta; e.g. one can look for a term

$$\lambda(0)\not{p}\lambda^*(p)\lambda^*(0)\not{p}\lambda(-p). \tag{3.8}$$

This term is generated by the operator $\lambda\not{\partial}\lambda^*\lambda^*\not{\partial}\lambda$, which is obtained from the θ integrations from eqn. (3.4). The coefficient of this operator is $g^2\frac{8}{6}\frac{1}{16\pi^2 M^4}$. The relative coefficients agree with what is expected from eqn. (3.4).

It is not difficult to generalize the term (3.1) to include the couplings of the hypermultiplets. One approach is to take this expression, written in $N=1$ language, and generalize it using the $SU(4)$ symmetry of the theory. In fact, it is enough to use the $SU(3)\times U(1)_R$ subgroup which is manifest in $N=1$ superspace. In $N=1$ language, the theory contains a vector multiplet (with $R=1$) and three chiral multiplets, Φ_i. These latter transform as a triplet of the $SU(3)$ with $R=2/3$. The $N=2$ vector multiplet contains one of these chiral fields, e.g. Φ_3. Writing out eqn. (3.1) in $N=1$ language, it is straightforward to generalize it so that it respects the full symmetry.

The effective action, eqn. (3.1), generalized as described above to include the couplings of the hypermultiplets, generates many terms including certain eight fermion operators with no derivatives. If one examines instanton effects in the theory, it might seems that these could generate eight fermion operators at zero momentum. An instanton in this theory possesses 16 fermion zero modes, before including the effects of the Higgs fields. Out of the 16 supercharges 8 annihilate the classical solution and the other 8 generate zero modes. Similarly, out of the 16 superconformal symmetries 8 annihilate the classical instanton configuration and 8 generate zero modes. If one proceeds as in instanton calcula-

tions in $N = 1$ supersymmetry [15] and in $N = 2$ supersymmetry [16], one can tie eight of the 16 fermion zero modes together with background scalars. It is easy to check that these terms are non-zero. If this were the whole story, it would violate the non-renormalization theorem we have just proven, so there must be some sort of cancellation. To see how this comes about, recall that the background Higgs fields are of order g. This is indeed what justifies instanton calculations, such as the calculation of baryon number violation in the standard model or the superpotentials and other quantities in supersymmetric theories [15,16]. But this means that the background fields are the same size as the fluctuating fields, so one must also consider the possibility of tying together the zero modes with scalar propagators. In $N = 1$ and $N = 2$ supersymmetric QCD, this is not possible in leading order. The only relevant couplings are the Yukawa couplings of the quarks to the gauginos (of the form $\phi^* \lambda \psi$), and the propagator $< \phi \phi >$ vanishes to lowest order. In the present case, however, one has not only these Yukawa couplings, but also Yukawa couplings arising from the superpotential, of the form $\phi \psi \psi$. As a result, there are now numerous diagrams, with various signs. We have not verified the cancellation explicitly, but similar cancellations have been seen in other contexts (e.g. ref. [17]) and we expect that they will occur here as well.

As we will discuss below, there is no possibility for an analogous cancellation in $d = 3$, so we do not expect that there is an exact non-renormalization theorem in this case.

4. Four derivative terms in $N = 8$ in $d = 3$

We now turn to the study of the $N = 8$ supersymmetric theory in three dimensions. For simplicity we consider only the gauge group $SU(2)$. The fields in the Lagrangian are three vector multiplets each of which consists of a vector, eight fermions and seven scalars. The global symmetry of the problem is $Spin(7)$. The eight supercharges transform as a spinor **8** of $Spin(7)$ and hence it is an R-symmetry. The seven scalars are in the vector **7** and the eight fermions in the spinor **8** of the symmetry group.

Along the flat directions the gauge $SU(2)$ symmetry is broken to $U(1)$ and the global $Spin(7)$ symmetry is broken to $Spin(6) \cong SU(4)$. The light fields are in one supermultiplet.

Since the long distance theory is free, we can dualize the vector in the multiplet to a compact scalar

$$\sigma \sim \sigma + 2\pi \tag{4.1}$$

and hence the moduli space is eight dimensional, parametrized by σ and seven scalars $\vec{\phi}$. It is

$$\mathcal{M} = \frac{\mathbb{R}^7 \times \mathbf{S}^1}{\mathbb{Z}_2} \tag{4.2}$$

where we mod out by $\mathbb{Z}_2$ because of the Weyl group of $SU(2)$. The metric on $\mathcal{M}$ is the obvious flat metric and has two orbifold singularities. The theory at the singularity $\vec{\phi} = \sigma = 0$ is likely to be interacting [8], while at $\vec{\phi} = 0$, $\sigma = \pi$, the theory is free – it is an orbifold theory [8].

The metric on the moduli space exhibits a global $U(1)$ symmetry of shifts along the $\mathbf{S}^1$ direction. It appears as a consequence of the Bianchi identity of the field strength, F, of the "photon" along the flat directions. This symmetry is not a symmetry of the microscopic theory. One way to see that is to note that in terms of the fundamental degrees of freedom F is a gauge invariant composite which does not satisfy the Bianchi identity. Alternatively, the magnetic monopoles of the four dimensional theory are instantons in three dimensions and they explicitly break the global $U(1)$ symmetry. These instantons were first studied in the theory without supersymmetry by Polyakov [18]. In theories with $N = 2$ supersymmetry they were discussed by Affleck, Harvey and Witten [19] and in $N = 4$ in [20,21]. More recently they were also discussed in theories with $N = 8$ supersymmetry [10,11]. Here we mention only the main properties of the instanton computation.

The expectation values of the scalar fields break the gauge symmetry to $U(1)$ and the global symmetry to $SU(4)$. In this set up there are BPS saturated field configurations which are monopoles in four dimensions and instantons in three. The instanton configuration breaks some of the remaining unbroken symmetries. For example, translation invariance in the three space-time dimensions is broken. For every such broken generator there is a collective coordinate which should be integrated over. Of particular interest are the fermion zero modes. They are also associated with symmetries of the problem. The sixteen supercharges can act on the classical configuration. Eight of them annihilate it and

therefore the instanton is BPS saturated. The other eight supercharges lead to fermion zero modes. Since they are associated with symmetry generators, one can introduce collective coordinates for them. It is easy to see that there are no other fermion zero modes.

The eight fermion zero modes transform as two **4**'s of the unbroken global $SU(4)$. They lead to an effective interaction which is a product of eight fermions

$$\prod_{I,A} \lambda^I_A \tag{4.3}$$

where $I = 1, ..., 4$ and $A = 1, 2$. Each fermion in this interaction transforms as $(\mathbf{4}, \mathbf{2})$ of $SU(4) \times SU(2)$ (the $SU(2)$ factor is the Euclidean space Lorentz group in three dimensions). The interaction (4.3) is $SU(4) \times SU(2)$ invariant. The full interaction term depends on the expectation values of the scalar fields. It includes a factor of

$$\exp\left(-\frac{< |\vec{\phi}| >}{g^2} + i\sigma\right). \tag{4.4}$$

The first term in the exponent $\frac{<|\vec{\phi}|>}{g^2}$ is the instanton action which depends on the scalar expectation value and the gauge coupling g. The second term $i\sigma$ shows that the instanton breaks the global $U(1)$ of shifts of σ by a constant. More factors of $< \vec{\phi} >$ are needed to make (4.3) not only $SU(4)$ invariant but $Spin(7)$ invariant.

Obviously, the full term we have just described is complex. In the action we have to add to it its hermitian conjugate. It is generated by an anti-instanton and involves the product of eight fermions transforming as $(\overline{\mathbf{4}}, \mathbf{2})$ of $SU(4) \times SU(2)$.

It is useful to compare this instanton computation to the analogous computation in four dimensions (section 3). In four dimensions the theory is conformally invariant and therefore there are sixteen fermion zero modes (eight from supersymmetry and eight from the superconformal symmetry). In three dimensions the theory is not conformally invariant and hence there are only eight zero modes. Furthermore, in three dimensions there is an exact instanton configuration which is BPS saturated (annihilated by half the supercharges). This is not true in four dimensions, where constrained instantons [22] have to be used. This makes the computation more delicate in four dimensions and leaves the possibility of the cancellation discussed in section 3.

We conclude that instantons generate terms with eight fermions. Therefore, by the discussion in the introduction they renormalize the terms in the effective action with eight fermions and also the terms with four derivatives. It is possible that these terms are also corrected by perturbative loop effects. Whether or not this is so, the non-renormalization theorem for these terms is not true in three dimensions.

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References

[1] N. Seiberg, "Naturalness Versus Supersymmetric Non-renormalization Theorems" Phys.Lett. **B318** (1993) 469, hep-ph/9309335.

[2] "The Power of Holomorphy, Exact Results in 4-D SUSY Field Theories," Talk given at Particles, Strings, and Cosmology (PASCOS 1994), Syracuse, New York, 19-24 May, 1994, hep-th/9408013

[3] B. de Wit, M.T. Grisaru and M. Rocek, Phys. Lett. **B374** (1996 297, hep-th/9601115; S.J. Gates, M.T. Grisaru, M. Rocek and W. Siegel, *Superspace* (Benjamin-Cummings 1983).

[4] N. Seiberg and E. Witten, "Electric-Magnetic Duality, Monopole Condensation, and Confinement in $N = 2$ Supersymmetric Yang-Mills Theory," Nucl.Phys. **B426** (1994) 19, hep-th/9407087.

[5] N. Seiberg and E. Witten, "Monopoles, Duality and Chiral Symmetry Breaking in N=2 Supersymmetric QCD," Nucl.Phys. **B431** (1994) 484, hep-th/9408099.

[6] M. Henningson, Nucl. Phys. **B458** (1996) 445, hep-th/9507135.

[7] F. Gonzalez-Rey, U. Lindstrom, M. Rocek, and R. von Unge, "On N=2 Low Energy Effective Actions," Phys. Lett. **B388** (1996) 581, hep-th/9607089

[8] N. Seiberg, "Notes on theories with 16 supercharges," RU-97-7, to appear.

[9] M. Green, J. Schwarz and E. Witten, *Superstring Theory*, Cambridge University Press, New York, 1987.

[10] J. Polchinski and P. Pouliot, "Membrane Scattering with M Momentum Transfer," NSF-ITP-97-27,hep-th/9704029.

[11] N. Dorey, V.V. Khoze and M.P. Mattis, "Multi-Instantons, Three-Dimensional Gauge Theory, and the Gauss-Bonnet-Chern Theorem," hep-th/9704197.

[12] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, "M Theory as a Matrix Model: A Conjecture," hep-th/9610043.

[13] J. Maldacena, RU-97-30, to appear.

[14] N. Dorey, V.V. Khoze and M.P. Mattis, "On Mass Deformed $N = 4$ Supersymmetric Yang-Mills Theory," hep-th/9612231.

[15] I. Affleck, M. Dine and N. Seiberg, "Dynamical Supersymmetry Breaking in Supersymmetric QCD," Nucl. Phys. **B241** (1984) 493.

[16] N. Seiberg, "Supersymmetry and Non-Perturbative Beta Functions," Phys.Lett. **206B** (1988) 75.

[17] D. Finnell and P. Pouliot, "Instanton Calculations Versus Exact Results in Four-Dimensional SUSY Gauge Theories," Nucl.Phys. **B453** (1995) 225, hep-th/9503115

[18] A.M. Polyakov, Nucl.Phys.**B120** (1977) 429.

[19] I. Affleck, J. Harvey and E. Witten, Nucl. Phys. **B206** (1982) 413.

[20] N. Seiberg and E. Witten,"Gauge Dynamics and Compactification to Three Dimensions, " IASSNS-HEP-96-78, hep-th/9607163.

[21] N. Dorey, V.V. Khoze, M.P. Mattis, D. Tong and S. Vandoren,"Instantons, Three-Dimensional Gauge Theory, and the Atiyah-Hitchin Manifold," hep-th/9703228.

[22] I. Affleck, Nucl.Phys. **B191** (1981) 429.